

Please check the examination details below before entering your candidate information

Candidate surname					Other names				
Centre Number				Candidate Number					
<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference

WMA13/01

Mathematics

International Advanced Level

Pure Mathematics P3

You must have:

Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

P71379A

©2022 Pearson Education Ltd.

1/1/1



Pearson

1. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = \frac{2x^3 - 4x - 15}{x^2 + 3x + 4}$$

- (a) Show that

$$f(x) \equiv Ax + B + \frac{C(2x + 3)}{x^2 + 3x + 4}$$

where A , B and C are integers to be found.

(4)

- (b) Hence, find

$$\int_3^5 f(x) dx$$

giving your answer in the form $p + \ln q$, where p and q are integers.

(5)

$$1. a) \quad f(x) = \frac{2x^3 - 4x - 15}{x^2 + 3x + 4}$$

$$\begin{array}{r} 2x - 6 \\ (x^2 + 3x + 4) \overline{) 2x^3 + 0x^2 - 4x - 15} \\ \underline{- 2x^3 + 6x^2 + 8x} \quad \vdots \\ -6x^2 - 12x - 15 \\ \underline{- -6x^2 - 18x - 24} \\ 6x + 9 \end{array}$$

$$\therefore \frac{2x^3 - 4x - 15}{x^2 + 3x + 4} = 2x - 6 + (6x + 9)$$

$$f(x) = 2x - 6 + \frac{3(2x + 3)}{x^2 + 3x + 4}$$

\downarrow \downarrow \downarrow
 $A=2$ $B=-6$ $C=3$



Question 1 continued

$$b) \int_3^5 f(x) \, dx$$

$$= \int_3^5 \frac{2x^3 - 4x - 15}{x^2 + 3x + 4} \, dx$$

$$= \int_3^5 2x - 6 + \frac{3(2x+3)}{x^2 + 3x + 4} \, dx$$

$$= [x^2 - 6x]_3^5 + \int_3^5 \frac{3(2x+3)}{x^2 + 3x + 4} \, dx$$

$$u = x^2 + 3x + 4$$

$$\frac{du}{dx} = 2x + 3 \rightarrow dx = \frac{du}{2x+3}$$

$$\int \frac{3(2x+3)}{u} \, dx = \int \frac{3(2x+3)}{u} \frac{du}{2x+3}$$

$$= \int \frac{3}{u} \, du = 3 \ln(u)$$

$$= 3 \ln(x^2 + 3x + 4)$$

$$= [x^2 - 6x]_3^5 + [3 \ln(x^2 + 3x + 4)]_3^5$$

$$= -5 + 9 + 3 \ln(44) - 3 \ln(22)$$

$$= 4 + 3 \ln\left(\frac{44}{22}\right)$$

$$\leftarrow \log_a b - \log_a c = \log_a \left(\frac{b}{c}\right)$$



blank

Question 1 continued

$$= 4 + 3 \ln(2)$$

$$= 4 + \ln(2^3)$$

$$= 4 + \ln(8)$$

$$\leftarrow a \log_b(c) = \log_b(c^a)$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



2. The functions f and g are defined by

$$f(x) = 5 - \frac{4}{3x+2} \quad x \geq 0$$

$$g(x) = \left| 4 \sin \left(\frac{x}{3} + \frac{\pi}{6} \right) \right| \quad x \in \mathbb{R}$$

(a) Find the range of f

(2)

(b) (i) Find $f^{-1}(x)$

(ii) Write down the domain of f^{-1}

(3)

(c) Find $fg(-\pi)$

(2)

$$2. a) \quad f(x) = 5 - \frac{4}{3x+2} \quad x \geq 0$$

$$\text{when } x=0 \rightarrow f(0) = 5 - \frac{4}{3(0)+2} = 3$$

$$\text{as } x \rightarrow \infty \rightarrow f(x) = 5 - \frac{4}{3x+2} \rightarrow 5 \quad \therefore f(x) \rightarrow 5$$

$3 \leq f(x) < 5$ $f(x)$ never actually = 5, only
 \uparrow tends to it \therefore not included in range

$$\text{to find } f^{-1}(x) : \quad f(x) = 5 - \frac{4}{3x+2}$$

① write the function using a "y"
and set equal to "x"

$$x = 5 - \frac{4}{3y+2}$$

② rearrange to make y the
subject

$$3xy + 2x = 15y + 10 - 4$$

$$y(3x-15) = -2x+6$$

③ replace y with $f^{-1}(x)$

$$y = \frac{6-2x}{3x-15}$$



Question 2 continued

$$\therefore f^{-1}(x) = \frac{6-2x}{3x-15}$$

(ii) domain of the inverse function :

↑ domain refers to the set of values we are allowed to plug into our function

domain of inverse function = range of function

↑ range refers to all possible values of a function

\therefore domain of $f^{-1}(x)$ = range of $f(x)$

$$\therefore 3 \leq x < 5$$

$$c) f(g(-\pi))$$

$$= f\left(\left|4 \sin\left(-\frac{\pi}{3} + \frac{\pi}{6}\right)\right|\right)$$

$$= f(2)$$

$$= \frac{5 - 4}{3(2) + 2} = \frac{1}{8}$$

Q2

(Total 7 marks)



3.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

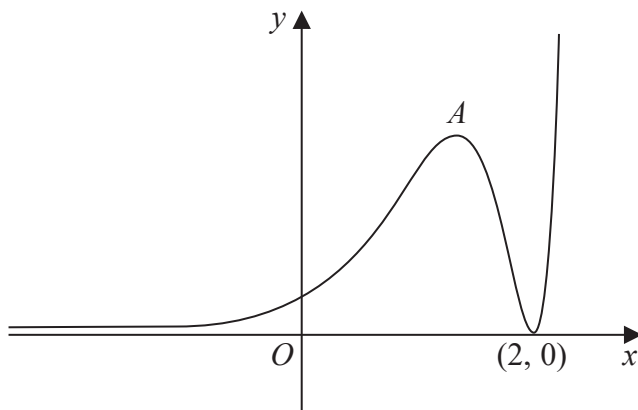


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = (x - 2)^2 e^{3x} \quad x \in \mathbb{R}$$

The curve has a maximum turning point at A and a minimum turning point at $(2, 0)$ (a) Use calculus to find the exact coordinates of A .

(5)

Given that the equation $f(x) = k$, where k is a constant, has **at least** two distinct roots,(b) state the range of possible values for k .

(2)

$$3. a) \text{ At max. } A \rightarrow \frac{dy}{dx} = 0$$

$$f(x) = (x-2)^2 e^{3x}$$

$$\text{PRODUCT RULE : } y = uv \quad y' = u'v + uv'$$

$$u = (x-2)^2$$

$$\frac{du}{dx} = 2(x-2)$$

$$v = e^{3x}$$

$$\frac{dv}{dx} = 3e^{3x}$$

$$f'(x) = (2(x-2))(e^{3x}) + ((x-2)^2)(3e^{3x})$$



Question 3 continued

$$= (x-2)(e^{3x})(2+3x-6)$$

$$= e^{3x}(x-2)(3x-4)$$

At turning points: $f'(x) = 0$

$$e^{3x}(x-2)(3x-4) = 0$$

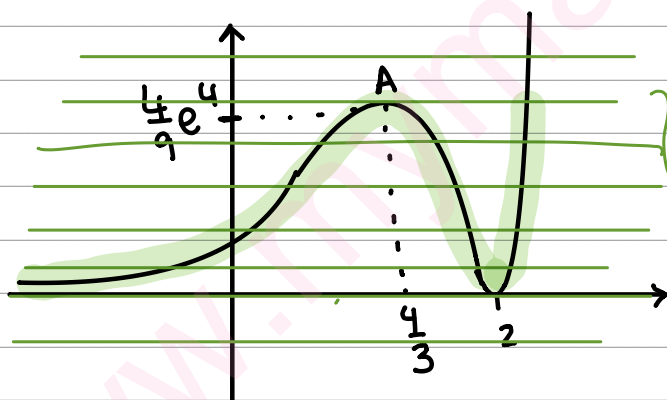
$$x = 2 \quad \cup \quad x = \frac{4}{3}$$

As we can see
from the graph, at $x=2$
there is a minimum

before

$$\therefore A = \left(\frac{4}{3}, \frac{4}{9}e^4\right)$$

b) if $f(x) = k$ has 2 distinct roots



represent different values
of k

We can see from the graph that there are 2
solutions when k is below / or at A
but above the x -axis

$$\therefore 0 < k \leq \frac{4}{9}e^4$$

Q3

(Total 7 marks)



4.

$$y = \log_{10}(2x + 1)$$

(a) Express x in terms of y .

(2)

(b) Hence, giving your answer in terms of x , find $\frac{dy}{dx}$

(3)

$$4.a) \quad y = \log_{10}(2x+1)$$

$$\text{LOG RULES} \rightarrow \log_a b = c \rightarrow a^c = b$$

$$10^y = 2x+1$$

$$x = \frac{1}{2}(10^y - 1)$$

$$b) \quad \frac{dy}{dx} = \frac{1}{(dx/dy)}$$

$$\text{given: } x = \frac{1}{2}(10^y - 1)$$

$$y = a^x \leftarrow \text{rewrite } a^x \text{ in terms of } e$$

$$a = e^{\ln(a)}$$

$$\downarrow \frac{d}{dx}(e^{kx}) = ke^{kx}$$

$$\therefore y = (e^{\ln(a)})^x \rightarrow \frac{dy}{dx} = \ln(a) \times e^{\ln(a)x} = \ln(a) \times a^x$$

$$\frac{dx}{dy} = \frac{1}{2} (\ln(10) \times 10^y)$$

$$\therefore \frac{dy}{dx} = \frac{1}{\frac{1}{2}(\ln(10) \times 10^y)} = \frac{2}{\ln(10) 10^y}$$



Question 4 continued

* We know that : $y = \log_{10}(2x+1)$

$$2x+1 = 10^y$$

$$\frac{dy}{dx} = \frac{2}{\ln(10)(2x+1)}$$

so $\frac{dy}{dx} = \frac{2}{(2x+1)\ln(10)}$

Q4

(Total 5 marks)



5.

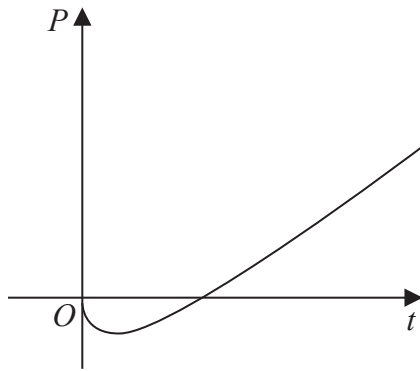


Figure 2

The profit made by a company, £ P million, t years after the company started trading, is modelled by the equation

$$P = \frac{4t - 1}{10} + \frac{3}{4} \ln \left[\frac{t + 1}{(2t + 1)^2} \right]$$

The graph of P against t is shown in Figure 2.

According to the model,

- (a) show that exactly one year after it started trading, the company had made a loss of approximately £830 000 (2)

A manager of the company wants to know the value of t for which $P = 0$

- (b) Show that this value of t occurs in the interval $[6, 7]$ (2)
- (c) Show that the equation $P = 0$ can be expressed in the form

$$t = \frac{1}{4} + \frac{15}{8} \ln \left[\frac{(2t + 1)^2}{t + 1} \right] \quad (2)$$

- (d) Using the iteration formula

$$t_{n+1} = \frac{1}{4} + \frac{15}{8} \ln \left[\frac{(2t_n + 1)^2}{t_n + 1} \right] \quad \text{with } t_1 = 6$$

find the value of t_2 and the value of t_6 , giving your answers to 3 decimal places. (3)

- (e) Hence find, according to the model, how many months it takes in total, from when the company started trading, for it to make a profit. (2)



Question 5 continued

5. a) when $t = 1$ year

$$P(1) = \frac{4(1) - 1}{10} + \frac{3}{4} \ln \left(\frac{1 + 1}{(2(1) + 1)^2} \right)$$

$$= \frac{3}{10} + \frac{3}{4} \ln \left(\frac{2}{9} \right)$$

$$= -0.828 \text{ million}$$

\therefore roughly profit after 1st year = -£830 000

So loss = £830 000

↑
negative sign
indicates money
was lost, \therefore loss

$$\left. \begin{array}{l} \text{b) } P(6) = -0.08799 \\ P(7) = 0.1975 \end{array} \right\} \begin{array}{l} \text{because } P \text{ is continuous} \\ \text{between } [6, 7] \end{array}$$

the sign change shows that P crosses the axis between $[6, 7]$ & so there is a root.

$$\text{c) } P = \frac{4t - 1}{10} + \frac{3}{4} \ln \left[\frac{t + 1}{(2t + 1)^2} \right] = 0$$

$$\frac{4t - 1}{10} = -\frac{3}{4} \ln \left[\frac{t + 1}{(2t + 1)^2} \right]$$

$$\frac{4t}{10} = -\frac{3}{4} \ln \left[\frac{t + 1}{(2t + 1)^2} \right] + \frac{1}{10}$$



Question 5 continued

$$t = -\frac{15}{8} \ln \left[\frac{t+1}{(2t+1)^2} \right] + \frac{1}{4}$$

LOG RULES $\rightarrow a \log_b(c) = \log_b(c^a)$

$$\therefore t = \frac{1}{4} + \frac{15}{8} \ln \left[\left(\frac{t+1}{(2t+1)^2} \right)^{-1} \right]$$

$$t = \frac{1}{4} + \frac{15}{8} \ln \left[\frac{(2t+1)^2}{t+1} \right]$$

$$d) t_{n+1} = \frac{1}{4} + \frac{15}{8} \ln \left[\frac{(2t_n+1)^2}{t_n+1} \right]$$

$$t_1 = 6$$

$$t_2 = t_{1+1} = \frac{1}{4} + \frac{15}{8} \ln \left[\frac{(2(6)+1)^2}{6+1} \right] = 6.220$$

$$t_3 = 6.287$$

$$t_5 = 6.312$$

$$t_4 = 6.307$$

$$t_6 = 6.314$$

e) From part (d), we know that the loss turns to positive profit at the point when $P = 0$

$$\therefore \text{when } t = 6.314 \text{ yrs}$$

$$\text{So months} = 6.314 \times 12 \approx 76 \text{ months}$$



6.

$$y = \frac{2 + 3 \sin x}{\cos x + \sin x}$$

Show that

$$\frac{dy}{dx} = \frac{a \tan x + b \sec x + c}{\sec x + 2 \sin x}$$

where a , b and c are integers to be found.

(6)

$$6. \quad y = \frac{2 + 3 \sin(x)}{\cos(x) + \sin(x)}$$

Quotient rule for differentiating : $y = \frac{u}{v} \rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

$$u = 2 + 3 \sin(x)$$

$$\frac{du}{dx} = 3 \cos(x)$$

$$v = \cos(x) + \sin(x)$$

$$\frac{dv}{dx} = -\sin(x) + \cos(x)$$

$$\frac{dy}{dx} = \frac{(\cos(x) + \sin(x))(3 \cos(x)) - (2 + 3 \sin(x))(-\sin(x) + \cos(x))}{(\cos(x) + \sin(x))^2}$$

$$= \frac{3 \cos^2(x) + 3 \sin(x) \cos(x) + 2 \sin(x) - 2 \cos(x) + 3 \sin^2(x) - 3 \sin(x) \cos(x)}{(\cos(x) + \sin(x))^2}$$

$$= \frac{3(\cos^2(x) + \sin^2(x)) + 2 \sin(x) - 2 \cos(x)}{(\cos(x) + \sin(x))^2}$$

$$\leftarrow \cos^2(A) + \sin^2(A) = 1$$

$$= \frac{3(1) + 2 \sin(x) - 2 \cos(x)}{\cos^2(x) + 2 \cos(x) \sin(x) + \sin^2(x)}$$

$$= \frac{3 + 2 \sin(x) - 2 \cos(x)}{1 + 2 \cos(x) \sin(x)}$$



Question 6 continued

(divide through by $\cos(x)$)

$$= \frac{3\sec(x) + 2\tan(x) - 2}{\sec(x) + 2\sin(x)}$$

$$a = 2$$

$$b = 3$$

$$c = -2$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



7.

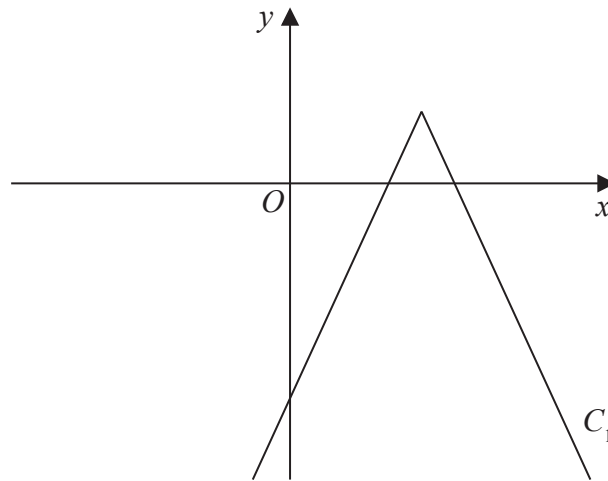


Figure 3

Figure 3 shows a sketch of the graph of C_1 with equation

$$y = 5 - |3x - 22|$$

(a) Write down the coordinates of

- (i) the vertex of C_1
- (ii) the intersection of C_1 with the y -axis.

(2)

(b) Find the x coordinates of the intersections of C_1 with the x -axis.

(2)

Diagram 1, shown on page 21, is a copy of Figure 3.

(c) On Diagram 1, sketch the curve C_2 with equation

$$y = \frac{1}{9}x^2 - 9$$

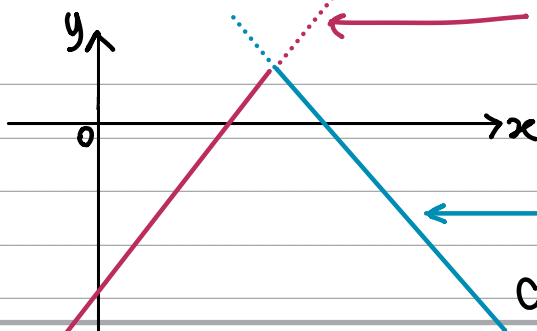
Identify clearly the coordinates of any points of intersection of C_2 with the coordinate axes.

(3)

(d) Find the coordinates of the points of intersection of C_1 and C_2
(Solutions relying entirely on calculator technology are not acceptable.)

(5)

7. a)



$y = 5 - (-(3x - 22))$
 $= 3x - 17$

$y = 5 - (3x - 22)$
 $= -3x + 27$



Question 7 continued

(i) B is the point at which the 2 lines meet

$$3x - 17 = -3x + 27$$

$$6x = 44 \rightarrow x = \frac{22}{3} \quad \therefore \text{vertex} = \left(\frac{22}{3}, 5\right)$$

(ii) We can see from the graph that the blue section intersects with the y-axis (y = 3x - 17)

 \therefore intersection of C_1 : (0, -17)b) We can see from the graph that both sections of C_1 intersect with the x-axis

intersection of pink section

$$0 = 3x - 17$$

$$x = \frac{17}{3}$$

intersection of blue section

$$0 = -3x + 27$$

$$x = \frac{27}{3} = 9$$

c)

$$C_2: y = \frac{x^2}{9} - 9$$

$$= \left(\frac{x}{3} - 3\right)\left(\frac{x}{3} + 3\right)$$

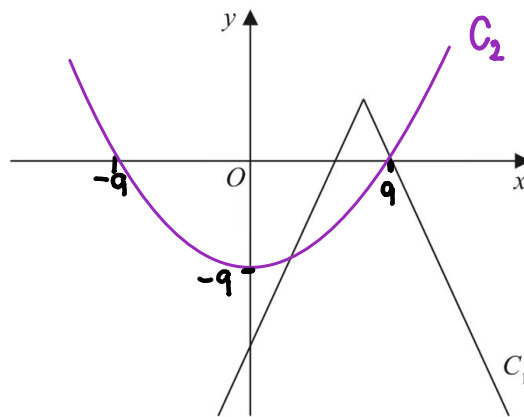
 \therefore roots of C_2 at 9 v -9

Diagram 1



Question 7 continued

e) C_2 intersects once with the pink section & the blue section

$$\frac{x^2 - 9}{9} = 3x - 17$$

$$x^2 - 27x + 72 = 0$$

$$(x - 24)(x - 3) = 0$$

$$\therefore x = \cancel{24} \cup x = 3$$

↑ reject as the pink section is

$$x < \frac{22}{3}$$

(i.e. on the left of the vertex)

$$24 > \frac{22}{3} \therefore \text{not solution}$$

\therefore points of intersection : $(3, -8)$
 $(9, 0)$

We can see from diagram 1 that

the blue section & C_2

intersect on the x -axis

at $(9, 0)$

Q7

(Total 12 marks)



8. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

- (a) Express $8 \sin x - 15 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R , and give the value of α , in radians, to 4 significant figures. (3)

$$f(x) = \frac{15}{41 + 16 \sin x - 30 \cos x} \quad x > 0$$

- (b) Find

(i) the minimum value of $f(x)$

(ii) the smallest value of x at which this minimum value occurs. (4)

- (c) State the y coordinate of the minimum points on the curve with equation

$$y = 2f(x) - 5 \quad x > 0 \quad (1)$$

- (d) State the smallest value of x at which a maximum point occurs for the curve with equation

$$y = -f(2x) \quad x > 0 \quad (1)$$

$$8. a) \text{ let : } 8 \sin(x) - 15 \cos(x) = R \sin(x - \alpha)$$

↑ using compound angle formulae
 $\sin(A - B)$
 $= \sin(A) \cos(B) - \sin(B) \cos(A)$

$$R(\sin x \cos \alpha - \sin \alpha \cos x) = 8 \sin(x) - 15 \cos(x)$$

↳ compare expanded expressions

$$R \sin x \cos \alpha - R \sin \alpha \cos x = 8 \sin(x) - 15 \cos(x)$$

$$R \cos \alpha = 8 \quad R \sin \alpha = 15$$



Question 8 continued

$$\frac{R \sin \alpha}{R \cos \alpha} = \tan \alpha = \frac{15}{8} \quad \left| \quad (R \cos \alpha)^2 + (R \sin \alpha)^2 \quad \begin{array}{l} \text{identity} \\ \cos^2 A + \sin^2 A = 1 \end{array} \right.$$

$$\alpha = 1.081$$

$$= R^2 (\cos^2 \alpha + \sin^2 \alpha) = R^2 (1)$$

$$= 8^2 + 15^2$$

$$= 289$$

$$\therefore R^2 = 289$$

$$\therefore \alpha = 1.081$$

$$R = 17$$

$$R = \pm 17$$

↑ reject negative solution
 $R > 0$

$$\text{b) (i) } f(x) = \frac{15}{41 + 16 \sin(x) - 30 \cos(x)}$$

$$= \frac{15}{41 + 2(8 \sin(x) - 15 \cos(x))}$$

$$= \frac{15}{41 + 2(17 \sin(x - 1.081))}$$

min value of $f(x)$ occurs when the denominator is the largest +

$$-1 \leq \sin(x - 1.081) \leq 1$$

$$7 \leq 41 + 2(17 \sin(x - 1.081)) \leq 75$$

$$\therefore \min f(x) = \frac{15}{41 + 2(17)} = \frac{1}{5}$$

↑ max value \therefore largest possible denominator



Question 8 continued

(ii) $\min f(x)$ occurs when :

$$\sin(x - 1.081) = 1$$

$$x - 1.081 = \frac{\pi}{2} \cup \dots$$

$$x = 2.65$$

c) $y = 2f(x) - 5 \quad x > 0$

↑
min(y) when min f(x)

$$\begin{aligned} \therefore y_{\min} &= 2f(x)_{\min} - 5 \\ &= 2\left(\frac{1}{5}\right) - 5 \\ &= -\frac{23}{5} \end{aligned}$$

d) $y = -f(2x)$

↑ reflection in the x-axis
↑ stretch scale factor $\frac{1}{2}$ parallel to x-axis

↳ so minimum of $f(x)$ becomes maximum of new graph

$$\therefore \text{value of } x = \frac{1}{2} \times 2.65 = 1.33$$



9. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Given that $\cos 2\theta - \sin 3\theta \neq 0$

(a) prove that

$$\frac{\cos^2 \theta}{\cos 2\theta - \sin 3\theta} \equiv \frac{1 + \sin \theta}{1 - 2\sin \theta - 4\sin^2 \theta} \quad (4)$$

(b) Hence solve, for $0 < \theta \leq 360^\circ$

$$\frac{\cos^2 \theta}{\cos 2\theta - \sin 3\theta} = 2 \operatorname{cosec} \theta$$

Give your answers to one decimal place.

(5)

$$9.a) \quad \text{LHS: } \frac{\cos^2(\theta)}{\cos(2\theta) - \sin(3\theta)} \quad \text{RHS: } \frac{1 + \sin(\theta)}{1 - 2\sin(\theta) - 4\sin^2(\theta)}$$

USING COMPOUND ANGLE FORMULAE $\rightarrow \sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\begin{aligned} \sin(3\theta) &= \sin(2\theta + \theta) = \sin(2\theta)\cos(\theta) + \cos(2\theta)\sin(\theta) \\ &= 2\sin(\theta)\cos^2(\theta) + (\cos^2(\theta) - \sin^2(\theta))\sin(\theta) \\ &= 2\sin(\theta)\cos^2(\theta) + \cos^2(\theta)\sin(\theta) - \sin^3(\theta) \\ &= 3\sin(\theta)\cos^2(\theta) - \sin^3(\theta) \end{aligned}$$



Question 9 continued

$$\therefore \text{LHS} : \frac{\cos^2(\theta)}{\cos^2(\theta) - \sin^2(\theta) - (3\sin(\theta)\cos^2(\theta) - \sin^3(\theta))}$$

write everything in terms of $\sin(\theta)$ \rightarrow $\frac{1 - \sin^2(\theta)}{1 - 2\sin^2(\theta) - 3\sin(\theta)(1 - \sin^2(\theta)) + \sin^3(\theta)}$

$\sin^2(A) + \cos^2(A) = 1$

$$= \frac{1 - \sin^2(\theta)}{1 - 3\sin(\theta) - 2\sin^2(\theta) + 4\sin^3(\theta)}$$

$$= \frac{(1 + \sin(\theta))\cancel{(1 - \sin(\theta))}}{\cancel{(1 - \sin(\theta))}(1 - 2\sin(\theta) - 4\sin^2(\theta))}$$

$$= \frac{1 + \sin(\theta)}{1 - 2\sin(\theta) - 4\sin^2(\theta)} = \text{RHS}$$

\therefore Identity proved

$$\text{b) } \frac{\cos^2(\theta)}{\cos(2\theta) - \sin(3\theta)} = \frac{1 + \sin(\theta)}{1 - 2\sin(\theta) - 4\sin^2(\theta)} = 2 \operatorname{cosec}(\theta)$$

$$\frac{1 + \sin(\theta)}{1 - 2\sin(\theta) - 4\sin^2(\theta)} = \frac{2}{\sin(\theta)}$$

$$\sin(\theta) + \sin^2(\theta) = 2 - 4\sin(\theta) - 8\sin^2(\theta)$$

$$9\sin^2(\theta) + 5\sin(\theta) - 2 = 0$$

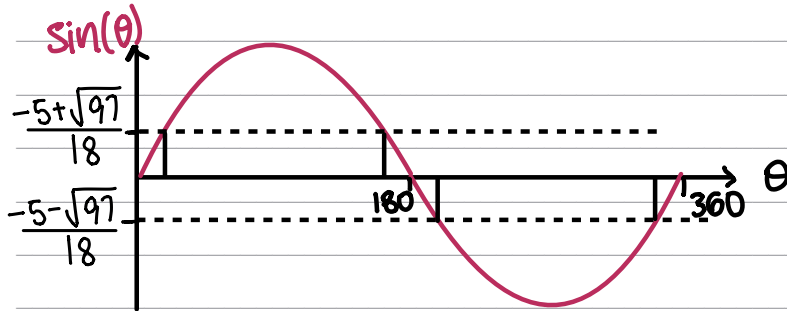
let $u = \sin(\theta)$

$$9u^2 + 5u - 2 = 0$$

$$u = \frac{-5 \pm \sqrt{97}}{18} \rightarrow \sin(\theta) = \frac{-5 \pm \sqrt{97}}{18}$$



Question 9 continued



$$0 < \theta \leq 360$$

↳ will be 4
values of θ in the
range

$$\theta = 15.6^\circ, 164.4^\circ, 235.6^\circ, 304.4^\circ$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

